Superselection rule in the one-dimensional hydrogen atom

1988 J. Phys. A: Math. Gen. 21 L651

(http://iopscience.iop.org/0305-4470/21/12/006)

J. Phys. A: Math. Gen. **21** (1988) L651–L653. Printed in the UK

LETTER TO THE EDITOR

Superselection rule in the one-dimensional hydrogen atom

H N Núñez Yépez†, C A Vargas†‡ and A L Salas Brito†‡

† Departamento de Física, Facultad de Ciencias, Universidad Nacional Autónoma de México, Apartado Postal 21-726, Coyoacan 04000, México DF, México
‡ Departamento de Ciencias Básicas, UAM-Azcapotzalco, Azcapotzalco 02200, México DF, México

Received 23 March 1988

Abstract. We show that a superselection rule operates between the bound states of the one-dimensional hydrogen atom and we investigate some of its consequences.

Several studies have recently appeared (Imbo and Sukhatme 1985, Tzara 1985, Núñez Yépez *et al* 1987, Davtyan *et al* 1987, Moss 1987, Núñez Yépez and Salas Brito 1987) of the quantum system described by the Hamiltonian (in atomic units: $\hbar = m = e = 1$)

$$\mathscr{H} = \tfrac{1}{2}p^2 - 1/|z| \tag{1}$$

i.e. the so-called one-dimensional hydrogen atom (1H). A peculiar feature of this system is its twofold degenerate discrete energy spectrum (Loudon 1959); such degeneracy has been attributed to the supersymmetric nature of the problem (Imbo and Sukhatme 1985) and has been explained by the mechanism of a hidden O(2) symmetry (Davtyan *et al* 1987). In this letter we will argue that the degeneracy can be explained by the operation of a superselection rule between the bound states of the 1H system; some consequences of the operation of this rule will be also discussed.

Two of the analyses of the 1H problem mentioned above have been performed in momentum space: Davtyan *et al* (1987) applied the Fock (1935) method to uncover the hidden symmetry of the problem, and in the process obtained both its bound and continuum state eigenfunctions; recently we (Núñez Yépez *et al* 1987) made a simpler analysis to obtain only its bound state eigenfunctions in momentum space. Comparing both papers, it can be easily seen that the bound state eigenfunctions obtained by Davtyan *et al* are merely even or odd superpositions of ours. But, despite this, both sets of eigenfunctions cannot be regarded as equivalent. The problem arises because Davtyan *et al* have not taken into account the restrictions in the domain of Hermiticity of $\mathscr{H}$ imposed by the singular nature of the $-1/|z|$ potential. These restrictions appear as follows: the Hamiltonian operator of the 1H problem is unbounded for every function not vanishing at $z = 0$; therefore it cannot be defined as Hermitian in the whole Hilbert space of square integrable functions (Akhiezer and Glazman 1961). For a function to be an eigenfunction of $\mathscr{H}$, it must vanish at the origin.

The restricted domain of Hermiticity of the Hamiltonian (1) has as an important consequence the existence of a superselection rule in the 1H system. This rule forbids the superposition of states on one side of the singularity of the potential with states on the other side. To prove this, note that the bound states of $\mathscr{H}$ are exactly the same as the $l = 0$ eigenstates of the three-dimensional Coulomb problem, either for $z > 0$ or $z < 0$. Let us denote these eigenstates as ψ_n^+ for $z > 0$, and as ψ_n^- for $z < 0$. If we form two arbitrary superpositions of these states

$$\phi_1 = a\psi_m^+ + b\psi_n^- \tag{2}$$

and

$$\phi_2 = b\psi_m^+ + c\psi_n^- \tag{3}$$

with $|a|^2 + |b|^2 = 1$ and $|b|^2 + |c|^2 = 1$, then the requirement that both of these states vanish at $z = 0$ implies that their Wronskian determinant must vanish too:

$$W(\phi_1, \phi_2) = 0. \tag{4}$$

Therefore, the functions ϕ_1 and ϕ_2 are not independent. If they are intended to represent physical states, one of the constants in the superpositions must vanish. To see this, let us take a Hermitian operator P with non-zero matrix elements between them—as the operator of inversion through the origin. If the constants in (2) and (3) were all different from zero, the expectation value of P between the states (2) would be in general different from its expectation value between the states (3), a result in contradiction with the implications of equation (4). We can say that the relative phase of ψ^+ and ψ^- in any superposition is of no physical consequence. Hence, we must conclude that there exists a superselection rule between the states ψ^+ and the states ψ^- which forbids coherent superpositions between them (Wick *et al* 1952, Bargmann 1959). Linear combinations of the ψ_n^+ with the ψ_m^- has meaning only as statistical mixed states. We must conclude also that the operator P is unobservable. A consequence of this is that, as we have pointed out before (Núñez Yépez *et al* 1987), the one-dimensional hydrogen atom has no even or odd eigenstates; the symmetry of inversion through the origin, obviously present in $\mathscr{H}$, is spontaneously broken. It must be clear now that the degeneracy in the discrete spectrum can be explained by the operation of the superselection rule.

It is somewhat surprising that a dynamically induced superselection rule (i.e. induced by specific non-kinematical properties of the Hamiltoian) operates in this system, for it is generally believed that this kind of superselection rule occurs only in complex systems with many degrees of freedom (Müller-Herold 1980, Pfeifer 1980, Zurek 1982). Our result shows that this is not necessarily the case, they can also occur in much simpler systems such as the 1H problem.

Notice that our argument rules out the possibility of the existence of the non-degenerate ground state of infinite energy predicted by Loudon (1959), a result that many authors still regard as correct (Moshinsky *et al* 1984, Imbo and Sukhatme 1985, Stedman 1985, Davtyan *et al* 1987, Moss 1987) despite the proofs that exist to the contrary (Andrews 1966, Núñez Yépez and Salas Brito 1987). The absence of a non-degenerate ground state implies (Gendenshtein and Krive 1985) the spontaneous breaking of supersymmetry in the one-dimensional hydrogen atom.

References

Akhiezer N I and Glazman M I 1961 *Theory of Linear Operators in Hilbert Space* vol 1 (New York: Ungar)
Andrews M 1966 *Am. J. Phys.* **34** 1194
Bargmann V 1959 *Ann. Math.* **59** 1

Davtyan L S, Pogosian G S, Sissakian A N and Ter-Antonyan V M 1987 *J. Phys. A: Math. Gen.* **20** 2765
Fock V A 1935 *Z. Phys.* **98** 145
Gendenshtein L E and Krive I V 1985 *Sov. Phys.- Usp.* **28** 645
Imbo T D and Sukhatme U P 1985 *Phys. Rev. Lett.* **54** 2184
Loudon R 1959 *Am. J. Phys.* **27** 649
Moshinsky M, Méndez N, Murow E and Hughes J W 1984 *Ann. Phys., NY* **155** 231
Moss R E 1987 *Am. J. Phys.* **55** 397
Müller-Herold U 1980 *Lett. Math. Phys.* **4** 45
Núñez Yépez H N and Salas Brito A L 1987 *Eur. J. Phys.* **8** 307
Núñez Yépez H N, Vargas C and Salas Brito A L 1987 *Eur. J. Phys.* **8** 189
Pfeifer P 1980 *Dissertation* ETH Zürich
Stedman G E 1985 *Eur. J. Phys.* **6** 225
Tzara C 1985 *Phys. Lett.* **111** 343
Wick G C, Wightman A S and Wigner E P 1952 *Phys. Rev.* **88** 101
Zurek W H 1982 *Phys. Rev.* D **26** 1862